

## GGSIPIU Mathematics 2004

1. If the angles between the pair of straight lines represented by the equation

$$x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0 \text{ is } \tan^{-1} \frac{1}{3}.$$

Where ' $\lambda$ ' is a non-negative real number, then  $\lambda$  is :

- a 2    b 0  
c 3    d 1

2. The distance of the line  $2x - 3y = 4$  from the point  $(1, 1)$  measured parallel to the line  $x + y = 1$  is :

- a  $\sqrt{2}$     b  $5/\sqrt{2}$   
c  $1/\sqrt{2}$     d 6

3. The equations of bisectors of the angles between the lines  $|x| = |y|$  are :

- a  $y = \pm x$  and  $x = 0$   
b  $x = \frac{1}{2}$  and  $y = \frac{1}{2}$   
c  $y = 0$  and  $x = 0$   
d none of these

4. The base of vertices of an isosceles triangle PQR are Q  $(1, 3)$  and R  $(-2, 7)$ . The vertex P can be :

- a  $(1, 6)$     b  $(\frac{1}{2}, 5)$   
c  $(\frac{5}{6}, 6)$     d none of these

5. The normal at the point  $(3, 4)$  on a circle cuts the circle at the point  $(-1, -2)$ . Then the equation of the circle is :

- a  $x^2 + y^2 + 2x - 2y - 13 = 0$   
b  $x^2 + y^2 - 2x - 2y - 11 = 0$   
c  $x^2 + y^2 - 2x + 2y + 12 = 0$   
d  $x^2 + y^2 - 2x - 2y + 14 = 0$

6. If  $\cos P = \frac{1}{7}$  and  $\cos Q = \frac{13}{14}$  where 'P' and 'Q' both are acute angles. Then the value of P-Q is :

- a  $30^\circ$     b  $60^\circ$

c  $45^\circ$       d  $75^\circ$

7. The equation  $3 \cos x + 4 \sin x = 6$  has ..... solution

a finite      b infinite

c one      d no

8. If  $\sec^{-1} x = \operatorname{cosec}^{-1} y$ , then  $\cos^{-1} \frac{1}{x} + \cos^{-1} \frac{1}{y}$  is equal to :

a  $\pi$       b  $\pi/4$

c  $-\pi/2$       d  $\pi/2$

9. If 'n' be any integer, then  $n^{n+1} 2^{n+1}$  is :

a odd number      b integral

multiple of 6

c perfect square      d does not

necessarily have any of the foregoing proof

10. If  $\tan \theta = -\frac{4}{3}$ , then the value of  $\sin \theta$  is :

a  $-\frac{4}{5}$  but  $\neq \frac{4}{5}$       b  $-\frac{4}{5}$  or  $\frac{4}{5}$

(  $\frac{4}{5}$  but  $\neq -\frac{4}{5}$       d  $\frac{1}{5}$

11. If  $c = 2 \cos \theta$ , then the value of the determinant  $\begin{vmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 6 & 1 & c \end{vmatrix}$  is :

a  $\frac{\sin 4 \theta}{\sin \theta}$       b  $\frac{2 \sin^2 2\theta}{\sin \theta}$

c  $4 \cos^2 \theta - 2 \cos \theta - 1$       d none of these

12. the set of values of x for which the inequality  $|x-1| + |x+1| < 4$  always holds true is :

a  $-2, 2$       b  $-\infty, 2 \cup$

$2, \infty$

c  $-\infty, 1] \cup [1, \infty$       d none

of these.

13. The equation of the parabola whose vertex is  $(-1, -2)$ , axis is vertical and which passes through the point  $(3, 6)$ , is :

a  $x^2 + 2x - 2y - 3 = 0$

b  $2x^2 = 3y$

c  $x^2 - 2x + 2y - 3 = 0$

d  $x^2 - 2x - 2y - 3 = 0$

14. The length of the axis of the conic  $9x^2 + 4y^2 - 6x + 4y + 1 = 0$  are :

a  $\frac{1}{2}, 9$       b  $3, \frac{2}{5}$

(c)  $\frac{2}{3}$       d  $3, 2$

15. If  $f(x) = \cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$  and  $g(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ , then  $\lim_{x \rightarrow a} \frac{f(x)-f(a)}{g(x)-g(a)}$ ,  $0 < a < \frac{1}{2}$ , is :

a  $\frac{3}{2(1+a^2)}$       b  $\frac{3}{2(1+x^2)}$

c  $\frac{3}{2}$       d  $-\frac{3}{2}$

16. If  $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2x - 1, & 1 < x \leq 1 \end{cases}$  then :

= 1

= 1

not differentiable at  $x = 1$

a  $f$  is discontinuous at  $x$

b  $f$  is differentiable at  $x$

c  $f$  is continuous but

d none of these

17.  $\lim_{x \rightarrow -2} \frac{\sin^{-1}(x+2)}{x^2+2x}$  is equal to :

x  $-2$

a 0      b  $\infty$

c  $-\frac{1}{2}$       d none of these

18. Let  $f(x) = x^p \cos \frac{1}{x}$ , when  $x \neq 0$  and  $f(x) = 0$ , when  $x = 0$ . then  $f(x)$  will be differentiable at  $x = 0$ , if :

- a  $p > 0$     b  $p > 1$   
 c  $0 < p < 1$     d  $\frac{1}{2} < p < 1$

1

19. The derivative of  $f(x) = 3|2+x|$  at the point  $x_0 = -3$  is :

- a 3            b -3  
 c 0            d none

of these

20. Derivative of the function  $f(x) = \log_5(\log_7 x)$ ,  $x > 7$  is :

- a  $\frac{1}{x \log_5(\log_7)(\log_7 x)}$   
 b  $\frac{1}{x(\log_5)(\log_7)}$   
 c  $\frac{1}{x \log x}$   
 d none of these

21. If  $z = x+iy$ ,  $z^{1/3} = a - ib$ , then  $\frac{x}{a} - \frac{y}{b} = k a^2 - b^2$ , where  $k$  is equal to :

- a 1            b 2  
 c 3            d 4

22. The number of real solutions of the equation  $1 + |e^x - 1| = e^x e^{-x} - 2$  is :

- a 1            b 2  
 c 4            d 8

23. The points of extrema of  $f(x) = \int_0^x \frac{\sin t}{t} dt$  in a domain  $x > 0$  are :

a  $2n+1 \frac{\pi}{2}$ ,  $n =$

1,2,.....

b  $4n+1 \frac{\pi}{2}$ ,  $n =$

1,2,.....

1,2,.....

c  $2n+1 \frac{\pi}{4}, n =$

d  $n \pi, n = 1, 2, \dots$

24. If  $x^2 + y^2 = t^2$  and  $x=s+3t, y=2s-t$ , then  $\frac{d^2u}{ds^2}$  is equal to :

a 12    b 10

c 32    d 36

25. If the equation  $x^2+qx+p = 0$  have a common root then  $p+q+1$  is equal to :

a 0    b 1

c 2    d -1

26. The value of  $a$  and  $b$  for which the sum of the cubes of the roots of  $x^2 - a - 2x + a - 3 = 0$  assumes the last value is :

a 3    b 4

c 5    d

none of these

27. Let  $z_1, z_2, z_3$  be three vertices of an equilateral triangle circumscribing the circle  $|z| = \frac{1}{2}$ . If  $z_1 = \frac{1}{2} + \frac{i\sqrt{3}}{2}$  and  $z_1, z_2, z_3$  were in anticlockwise sense, then  $z_2$  is :

a  $1 + \overline{3i}$     b

1-  $\overline{3i}$

c 1    d -1

28. If  $z = \frac{-2}{1 + \sqrt{3}i}$ , then the value of  $\arg z$  is :

a  $\pi$     b

$\pi/3$

c  $2\pi/3$     d

$\pi/4$

29. Let  $\omega$  is an imaginary cube roots of unity ,then the value of

$21 + \omega + \omega^2 + 32 + \omega + 12 + \omega^2 + 1 + \dots + n + 1 + n + \omega + 1 + n + \omega^2 + 1$  is :

- a  $\left[\frac{n(n+1)}{2}\right]^2 + n \left[\frac{n^2(n+1)^2}{4}\right]$   
 c  $\left[\frac{n(n+1)}{2}\right]^2 - n$  d none of these

30. The locus of the point z satisfying  $\arg\left(\frac{z-1}{z+1}\right) = k$ , (where k is non zero) is :

- a a circle with centre on y-axis  
 b circle with centre on x-axis  
 c a straight line parallel to x-axis  
 d a straight line making an angle  $60^\circ$  with the x-axis

31. If  $P(3,4,5), Q(4,6,3), R(-1,2,4), S(1,0,5)$ , then the projection of RS on PQ is :

- a  $-\frac{2}{3}$  b  $-\frac{4}{3}$   
 c  $\frac{1}{2}$  d 2

32. If a line makes  $\alpha, \beta, \gamma$  with the positive direction of x, y, z-axes respectively. Then  $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$  is equal to :

- a  $\frac{1}{2}$  b  $-\frac{1}{2}$   
 c -1 d 1

33. The projection of a line on co-ordinate axes are 2, 3, 6. Then the length of the line is :

- a 7 b 5  
 c 1 d 11

34. The decimal equivalent of the binary number 10011.1 is :

- a 19.50 b 11001.11  
 c 5005.55  
 d 19.10

35. The binary represents of 60 is :

111100

a 101110 b

110000

c 110011 d

36. Which of the following statement is not tautology ?

b  $p \vee q \wedge p$

a  $\sim p \vee q \wedge p$

d  $\sim p \vee q \wedge \sim p \wedge p$

c  $q \vee \sim p \vee q$

37. The period of  $f(x) = \sin\left(\frac{rx}{n-1}\right) + \cos\left(\frac{rx}{n}\right)$ ,  $n \in \mathbb{Z}, n > 2$  is :

b  $4\pi n - 1$

a  $2\pi n - 1$

d none of these

c  $2\pi n - 1$

39. The radius of the circle whose arc of length 15 km makes an angle of  $\frac{3}{4}$  radian at the centre, is :

b 20 cm

a 10 cm

d  $22 \frac{1}{2}$  cm

c  $11 \frac{1}{4}$  cm

40. If  $f_n(x) = e^{f_{n-1}(x)}$ , for all  $n \in \mathbb{N}$  and  $f_0(x) = x$ , then  $\frac{d}{dx}\{f_n(x)\}$  is equal to :

b  $f_n(x) \cdot \frac{d}{dx}\{f_{n-1}(x)\}$

a  $f_n(x) \cdot f_{n-1}(x)$

.... $f_2(x) \cdot f_1(x)$  d none of these

c  $f_n(x) \cdot f_{n-1}(x)$

41. if  $3^x + 2^{2x} \geq 5^x$ , then the solution set for x is :

b  $[2, \infty)$

a  $[-\infty, 2]$

d {2}

c [0,2]

42. The number of integral solution of  $\frac{x+1}{x^2+2} > \frac{1}{4}$  is :

b 2

a 1

d none of these

c 5

43. The value of k for which the equation  $k - 2x^2 + 8x + k + 4 = 0$  has both real, distinct and -ve, is :

b 2

a 0

d -4

c 3

44. The triangle PQR of which the angles P, Q, R satisfy  $\cos P = \frac{\sin Q}{2 \sin R}$  :

b right angled

a equilateral

d isosceles

c any triangle

45. If  $f(x) = a - x^{n-1/n}$ , where  $a > 0$  and  $n$  is a positive integer, then  $f[f(x)]$  is equal to :

b  $x^{-2}$

a  $x^{-3}$

d none of these

c  $x$

46. The function  $f(x) = [x]^2 - [x^2]$  where  $[y]$  is the greatest integer less than or equal to  $y$  is discontinuous at :

except 0 and 1

a all integers

b all integers



except 0

c all integers

except 1

d all integers

47. the function  $f(x) = |px - q| + r|x|$ ,  $x \in -\infty, \infty$  where  $p > 0, q, r > 0$  assumes its maximum value only at one point, if :

$q \neq r$

a  $p \neq q$       b

$p = q = r$

c  $r \neq p$       d

48. A function  $f(x) = \frac{x^2 - 3x + 2}{x^2 + 2x - 3}$  is :

$x = -3$

a maximum at

at  $x = -3$  and maximum at  $x = 1$

b maximum

$x = 1$

c maximum at

increasing in its domain

d function is

49. The locus of the point  $(x, y)$  satisfying the relation

$$\sqrt{(x-3)^2 + (y-1)^2} + \sqrt{(x+3)^2 + (y-1)^2} = 6 \text{ is}$$

- a Straight line
- b Pair of straight lines
- c Circle
- d Ellipse

50. If  $z_1, z_2$  and  $z_3$  are complex number such that  $|z_1| = |z_2| = |z_3| = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 1$  then  $|z_1 + z_2 + z_3|$  is :

a equal to 1

b less than 1

than 3

c greater

d equal to 3

51. Let  $a_1, a_2, a_3$  be any positive real numbers, then which of the following statement is not true ?

$a_1^3 + a_2^3 + a_3^3$

a  $3a_1 a_2 a_3 \leq$

$\frac{a_3}{a_1} \geq 3$

b  $\frac{a_1}{a_2} + \frac{a_2}{a_3} +$

$\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right) \geq 9$

c  $a_1 a_2 a_3$

$\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)^3 \leq 27$

d  $a_1 a_2 a_3$

52. If  $ab = 2a + 3b, a > 0, b < 0$ , then the minimum value of  $ab$  is :

b 24

a 12

d none of these

c  $\frac{1}{4}$

53. Let  $N$  be +ve integer  $\neq 1$ , then none of the numbers  $2, 3, \dots, N$  is divisor of  $N! - 1$ . So we can conclude that  $N! - 1$  is :

number

a prime

one of this number  $N+1, N+2, \dots, N! - 2$  is divisor of  $N! - 1$

b at least

smallest numbers between  $N$  and  $N!$  which is divisor of  $N! - 1$  is prime number

c The

these

d none of

54. If  $f(x) = \cos[\pi^2 x] + \cos[-\pi^2 x]$ , then :

a  $f(\pi/4) = 2$

- b  $f^{-\pi} = 2$
- c  $f^{\pi} = 1$
- d  $f^{\pi/2} = -1$

55. Let  $f(x) = \frac{x^2 - 4}{x^2 + 4}$ , for  $|x| > 2$ , then the function  $f: (-\infty, -2] \cup [2, \infty) \rightarrow [-1, 1]$  is :

- a one - one into
- b one - one onto
- c many - one into
- d many - one onto

56. The function  $f(x) = \sin \log x + \sqrt{x^2 + 1}$  is :

- a even function
- b odd function
- c even nor odd
- d periodic function

57. The range of  $f(x) = \sec\left(\frac{\pi}{4} \cos^2 x\right)$ ,  $-\infty < x < \infty$  is :

- a  $[1, \sqrt{2}]$
- b  $[1, \infty)$
- c  $[-\sqrt{2}, -1]$
- d  $(-\infty, 1] \cup [1, \infty)$

58. For any three sets  $A_1, A_2, A_3$ . Let  $B_1 = A_1, B_2 = A_2 - A_1$  and  $B_3 = A_3 - A_1 \cup A_2$ , then which of the following statement is always true ?

- a  $A_2 \cup A_3 \supset B_1 \cup B_2 \cup B_3$
- b  $A_2 \cup A_3 = B_1 \cup B_2 \cup B_3$
- c  $A_2 \cup A_3 \subset B_1 \cup B_2 \cup B_3$
- d none of these

59. The domain of the function  $f(x) = \frac{\sin^{-1}(3-x)}{\log(x-2)}$  is :

**b 3,4]**

**a [2,4]**

**d  $-\infty, 3 \cup [2, \infty$**

**c [2,  $\infty$**

**60. The remainder obtained when  $1! + 2! + \dots + 200!$  is divided by 14 is :**

**b 4**

**a 3**

**d none of these**

**c 5**